

COOLING OF THE HOT REGION FORMED
BY THE BREAKDOWN OF AIR WITH
LASER RADIATION

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The problem is solved of the cooling of the hot spherical region ("fireball") formed by a focused laser explosion in air at an energy of 5.3 J. The true spectral composition of the radiation and its directivity diagram was considered. Table values of the absorption coefficients were used. The temperature and density distributions in the fireball are found for a specified law of the change of pressure inside it (taken from the solution of the corresponding gas-dynamic problem without radiation). The appearance of sharp regions of temperature change are noted — temperature steps associated with a change of the degree of ionization and with the corresponding change of the spectral coefficients of absorption of air. The spectral and total radiation fluxes are determined. The fraction of de-excited energy (of the total) is computed as a function of time. The phenomenon of breakdown originates with powerful focusing of laser radiation in gases. A review of the appropriate papers and a description of the phenomenon are given in [1, 2].

In consequence of the high concentration of energy, the gas in the region of a breakdown heats up to extremely high temperatures. The pressures are considerable. Thus, according to a calculation carried out by the usual relations and similar to [1, 2] but with a real equation of state for air [3], and with a radiation flux density of $F = 0.5 \cdot 10^4$ MW/cm², the detonation velocity $D = 35$ km/sec, the pressure behind the detonation wave $p_d = 6530$ kg/cm², and the temperature $T_d = 10^5$ °K. Thus, with this radiation flux density, the temperature and pressure are found to be of the same order as in the fireball of a nuclear explosion at the instant of formation of the shock wave from the thermal wave [2, 4, 5].

The fireball from a laser explosion in the stage of energy dissipation, generally speaking, does not possess spherical symmetry [6], except in cases of specially created spherically symmetrical irradiation [7].

The region of the explosion usually is elongated along the direction of propagation of the radiation. However, the asymmetry is small. During the time of action of the laser, of order 10^{-8} sec, the detonation wave traverses 10^{-1} cm with a front velocity of $\sim 10^7$ cm/sec, while a typical value of the radius of the beam in the region of maximum focusing amounts to only 10^{-2} cm. Moreover, owing to the high pressures in the region of energy release, a shock wave is propagated through the gas in a direction perpendicular to the beam, which reduces the asymmetry.

In the observations in [8], the shock wave from a laser explosion was recorded during 35–430 nsec after the explosion in the form of an ellipse with a ratio of 2:1.5 for the semiaxes.

The authors have carried out controlled calculations of the cooling of a cylinder with a height to radius ratio of 3:1 by a two-dimensional difference method, similar to the one-dimensional method described below. Comparison of this calculation with the spherically symmetrical one, with identical total initial explosion energy, indicated that the divergence of values of the temperature at the center, the fraction of energy dissipated, and other integral characteristics at one and the same instant of time did not exceed 5%. More-

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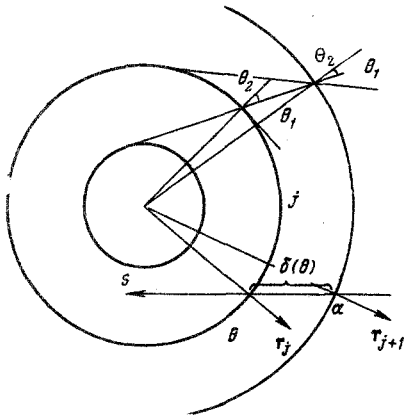


Fig. 1

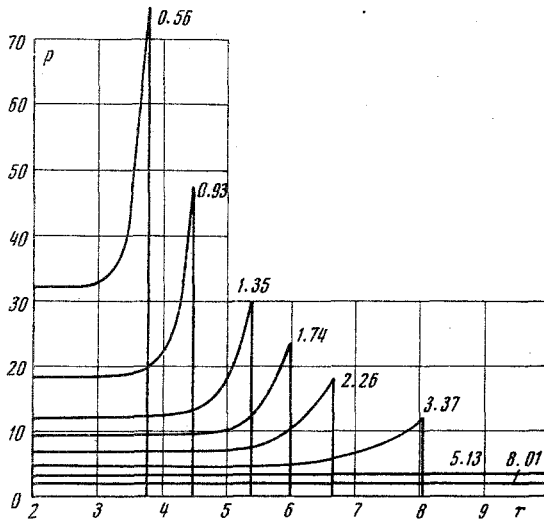


Fig. 2

over, asymmetry is observed at the initial instants of time when the dissipation is still small (as the calculations show, the characteristic time of de-excitation is of the order of tens and hundreds of microseconds).

At this stage, after completion of the effect of the radiation, the pattern of motion of the substance is close to the pattern of a powerful explosion. Sonic perturbations balance out the pressure inside the hot core, and the velocity of the shock wave in all directions becomes identical and the fireball becomes spherically symmetrical. If we assume that the adiabatic index is constant, then the expansion of the fireball in the energy phase is described, to a first approximation, by the self-similar solution of the problem of a point explosion [9].

In order to obtain a more accurate solution, the back pressure and variability of the adiabatic index must be taken into account. The distribution of the parameters can be found numerically by the methods in [10, 11], taking into account the actual equation of state of air [2, 3].

Measurement of the shock wave parameters at large distances from the center of the "explosion" permits the energy release to be determined. However, the problem arises concerning what fraction of the energy of a laser explosion is carried by the radiation. For large-scale explosions, of order 10^{12} - 10^{16} J, the pattern of the light phenomena is described in [2, 4, 5]. In these cases, about 30% of the total energy is emitted, mainly at the stage when the fireball has expanded almost to its maximum dimensions, determined in practice by gas-dynamic processes, and the temperature of the gas behind the shock wave has dropped so much that the gas has become transparent - "breakaway" of the shock wave front from the fireball has taken place. The energy of a laser explosion is considerably less than that from a nuclear explosion -

a total of only a few joules or a few tens of joules. The characteristic dimensions of the fireball also are correspondingly less - by four to five orders of magnitude. At the same time, the magnitudes of the spectral ranges (mean free paths) of the radiation at the same temperature and density, i.e., at corresponding instants of time, remain unchanged.

The interesting problem is that a change of optical thickness of the fireball does not lead to a sharp change of optical phenomena and a change of the fraction of the emitted energy.

The problem is solved by numerical methods in this paper. Simultaneously, the magnitudes of the intensity and flux of the radiation emerging from the fireball is determined successfully; i.e., the characteristics are determined of a laser explosion as a source of radiation in the optical and UV range, which is of independent interest.

1. The solution of the whole problem of the motion of a substance, taking into account radiation transfer of a continuous spectrum, even in the case of spherical symmetry, presents considerable difficulties, including those of a "technological" order as, in addition to considering the gas-dynamic equations at each step in time, a repeated calculation of the transport equations is required for all selected directions and frequencies of the radiation.

The use of methods of accurate averaging of the transport equation at certain instants of time obviously can be a considerable facilitation [12]. As the nature of the temperature and density distributions during explosion change only weakly, averaging of the transport equation probably can be carried out relatively infrequently.

In the first stage of solving the problem, the fact that emission processes may affect relatively weakly the magnitude of the pressure in the fireball can also be taken into account even in the case of a quite power-

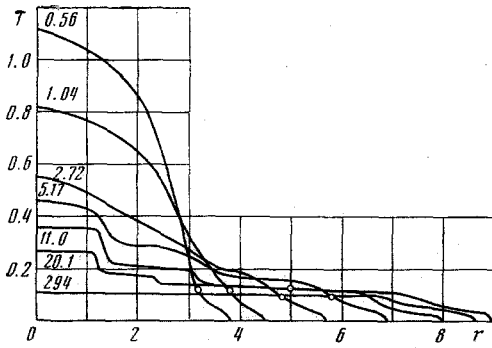


Fig. 3

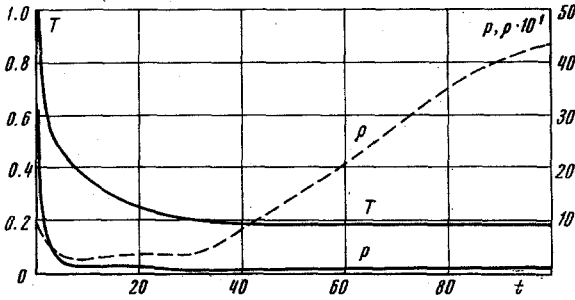


Fig. 4

ful effect of radiation on other physical parameters (internal energy, temperature, density). Actually, at the start of the process of fireball growth, when a large drop in pressure occurs at the shock wave front, the energy losses by radiation are negligibly small. At the time when the energy losses by radiation become significant, the pressure inside the fireball is equalized, having approached atmospheric pressure, so that radiation cannot have any significant effect on the magnitude of the pressure. These qualitative considerations are confirmed by the results of [13], in which calculations were carried out for an explosion with and without taking account of radiation. In both cases, close values are obtained for the magnitude of the pressure.

The assumption concerning the weak effect of radiation on the magnitude of the pressure highly simplifies the problem, as it permits the pressure to be calculated even by the well-known function obtained from the solution of the gas-dynamic equation, without taking radiation into account or with an approximate estimate of the radiation.

In order to calculate the cooling of the fireball with a specified law of change of pressure and known thermodynamic and optical properties of the gas, it is necessary jointly to consider only the equation of conservation of energy and radiation transfer [14].

The equation of conservation of energy can be written in the form

$$\rho \frac{\partial H}{\partial t} - \frac{\partial p}{\partial t} = - \operatorname{div} F \quad (1.1)$$

where ρ is the density of the medium, H is the thermal energy of unit mass, p is the pressure, and F is the flux density of the radiant energy.

The transport equation for a spectral radiation intensity I_ν , propagating in the direction S , in the case of quite slow processes [when the term $c^{-1}(\partial I_\nu / \partial t)$ can be neglected, where c is the velocity of light] has the form

$$\frac{\partial I_\nu}{\partial S} = k_\nu (B_\nu - I_\nu) \quad \left(B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \right) \quad (1.2)$$

Here k_ν is the linear coefficient of absorption of the substance for quanta with radiation frequency ν , B_ν is the Planck equilibrium function, T is the temperature, and k is Boltzmann's constant.

The radiant flux density F has the form

$$F = 2\pi \int_0^\pi \int_0^\infty I_\nu d\nu \cos \theta d \cos \theta \quad (1.3)$$

where θ is the angle formed by the direction of propagation of the radiation S and the radius vector r .

The system of equations (1.1)-(1.3), together with the known equation of state of the gas

$$H = H(p, T), \quad \rho = \rho(p, T) \quad (1.4)$$

and the spectral coefficients of absorption $k_\nu = k_\nu(p, T)$ given by the tables, is a closed integral-differential system of equations. In this paper, this system of equations has been solved numerically without any simplifying assumptions about the nature of the spectrum and the directivity diagram of the radiation.

In order to integrate this system of equations, the method of subdividing the region into layers was used whereby the problem is reduced to a system of ordinary differential equations for each layer. This method for solving gas-dynamic problems was used in [15].

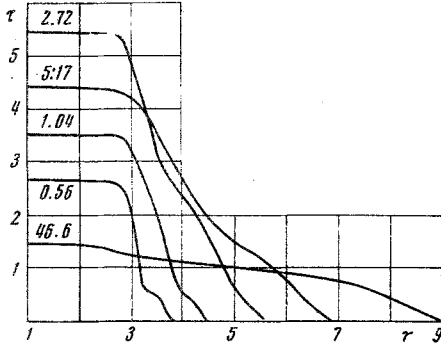


Fig. 5

We shall now summarize the scheme of the calculation for the system of equations (1.1)-(1.4).

2. A spherical zone of the hot gas is subdivided into spherical layers (Fig. 1). The state of each layer j bounded by radii r_j and r_{j+1} at every instant of time t defines the following quantities averaged over the layer: H_j is the enthalpy; p_j is the pressure; ρ_j is the density; T_j is the temperature; and $k_{\nu,j}$ is a set of spectral coefficients of absorption for given values of T_j and p_j (ν is the frequency index).

It is assumed that inside each layer the spectral coefficients are constant with respect to radius and that the temperature is varying according to a parabola. The Planck function in the layer is given in the form

$$B_{\nu}(r) = a_{\nu,j} + b_{\nu,j}r^2 \quad (2.1)$$

where $a_{\nu,j}$ and $b_{\nu,j}$ are constants defined by the Planck function $B_{\nu,j}$ and $B_{\nu,j+1}$, which correspond to the temperatures at the boundaries of the layer

$$b_{\nu,j} = (B_{\nu,j+1} - B_{\nu,j}) / (r_{j+1}^2 - r_j^2) \quad (2.2)$$

With these assumptions, the radiation transport equation (1.2) is integrated. The spectral intensity of the radiation at the point r_j in the direction forming an angle θ with the radius vector at this point is written in the form (we omit the frequency index ν)

$$I_j(\theta) = I_s(\alpha) e^{-k_i \delta(\theta)} + B_j - B_s e^{-k_i \delta(\theta)} - \frac{2b_i}{k_i} \left\{ \left(r_j \cos \theta - \frac{1}{k_i} \right) (1 - e^{-k_i \delta(\theta)}) + \delta(\theta) e^{-k_i \delta(\theta)} \right\} \quad (2.3)$$

Here α is the angle at which the radiation must leave the point r_S in order to arrive at the point r_j at the angle θ ; $\delta(\theta)$ is the path traversed by the radiation in the layer i along the ray forming the angle θ with the radius-vector at the point r_j .

$$|\cos \alpha| = \sqrt{1 - (r_j/r_S)^2 \sin^2 \theta}, \quad \delta(\theta) = r_j \cos \theta - r_S \cos \alpha \quad (2.4)$$

Depending on the size of the angle θ , the indexes S and i assume the following values:

$$S = \begin{cases} j-1 & \text{for } \theta \leq \theta_2 < \theta_2, \quad (\theta_2 = \arcsin(r_{j-1}/r_j)) \\ j & \text{for } \theta_2 \leq \theta < \pi/2 \\ j+1 & \text{for } \pi/2 \leq \theta \leq \pi \end{cases} \quad (2.5)$$

$$i = \begin{cases} j-1 & \text{for } 0 \leq \theta < \pi/2 \\ j & \text{for } \pi/2 \leq \theta \leq \pi \end{cases}$$

The direction forming an angle θ_2 with the radius-vector at the point r_j is the tangent to the boundary surface of the adjacent (inside) layer.

In calculating the radiation field, the spectral intensities must be determined at every point of the given set of radii for all fixed angles θ ($0 \leq \theta \leq \pi$), formed by the direction of the radiation with the radius-vector at every point.

If the intensity of the incident radiation is known at the boundary of the region defined by the radius R (in the case being considered, it is equal to zero, $I_{\nu}(R, t, \theta) = 0$ when $\pi/2 \leq \theta \leq \pi$), then according to Eq. (2.3)-(2.5), it is possible, proceeding from the outside layers to the inside layers, to determine successively the intensity of radiation at all points r_j for the angles $\pi/2 \leq \theta \leq \pi$.

The values obtained for the spectral intensity at any point r_j for the angles $\pi/2 \leq \theta \leq \pi$ are simultaneously the starting values for finding the radiation intensity at the same point r_j in the reverse direction over the range of angles $\theta_2 \leq \theta < \pi/2$ (the case $S = j$) which, in their turn, are the starting values for the successive determination of the radiation intensities at all points with radii $r_p \geq r_j$ ($j \leq p \leq j_m + 1$, where j_m is a whole number of layers) in the angular range $\theta_2 \leq \theta < \theta_1$, where

$$\cos \theta_1 = \sqrt{1 - (r_j/r_p)^2}, \quad \cos \theta_2 = \sqrt{1 - (r_{j-1}/r_p)^2} \quad (2.6)$$

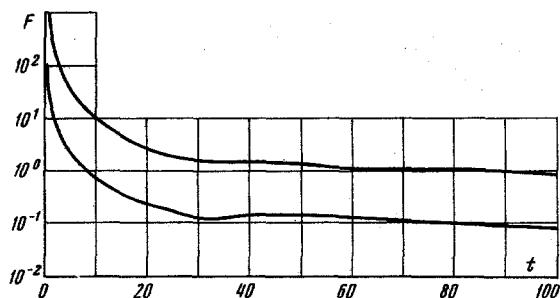


Fig. 6

When $p=j$, the angle $\theta_1 = \pi/2$, in accordance with Eq. (2.5). The radiation intensity in the radial direction (angle $\theta = 0$) at any point can be obtained if the radiation intensities at the outer boundary of the central layer with radius $r_1 = 0$ and r_2 are calculated. In this case $\theta_2 = 0$ for all radii $r_2 \leq r_j \leq R$.

The proposed scheme of calculation of the transport equation permits the volume of computer storage necessary for a precise calculation to be reduced considerably, as in this case the total (with respect to spectrum and angles) set of spectral intensities is stored only for a single point of the radius; for all the remaining points along the radius, the values integrated over the spectrum and angle — the radiation fluxes — are stored.

The radiation flux densities, calculated by Eq. (1.3), at every point are substituted in the energy equation (1.1).

Euler's method with recalculation was used for solving the energy equation.

3. The results are presented in this paper of a calculation of a laser explosion with a total energy of 5.3 J at normal initial air density ($\rho_0 = 1.29 \cdot 10^{-3} \text{ g/cm}^3$).

The instant $t_0 = 0.56 \mu\text{sec}$ was taken as the starting time for the calculation, which is close to the instant of breakaway of the shock front from the fireball.

The distribution of the physical parameters [temperature — for this instant of time, and pressure — for different instants of time (Fig. 2)] are obtained by the method described in [11], but taking account of the actual equation of state of air.

Tabular values were used in the calculation for the equation of state [3] and the spectral absorption coefficients of air over a wide frequency range ($0.06 \text{ eV} \leq h\nu \leq 250 \text{ eV}$) [16]. For temperatures of $T > 20,000^\circ\text{K}$, the absorption coefficients were calculated by Yu. P. Vysotskii and V. A. Nuzhnii by the procedure in [16], but without taking account of absorption lines.

At the start of the calculation, the heated volume was subdivided into 20-30 spherical layers with a temperature drop between adjacent layers of not more than 10%. The scheme of the calculation was designed to include additional computational points along the radius if the temperature drop exceeded 5-10%. The number of points over angles in the range $0 \leq \theta \leq 180^\circ$ amounted to 40-50.

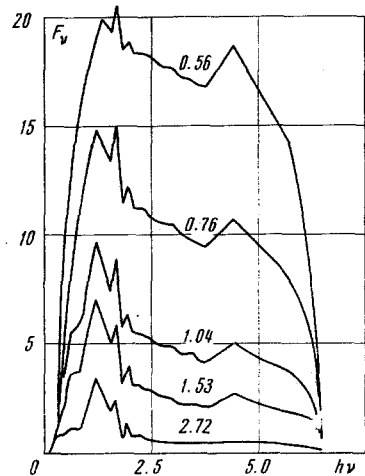


Fig. 7

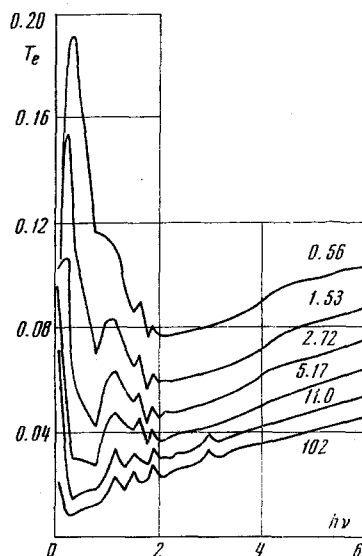


Fig. 8

A twofold increase in the number of points with respect to radius and angle changed such parameters as the fraction of emitted energy and the temperature at the center by not more than 1%.

The number of spectral intervals was chosen equal to 60-80, so that approximately one-half of them fitted into the spectral range from 6 to 7 eV, in which energy emission takes place outwards. Their bound-

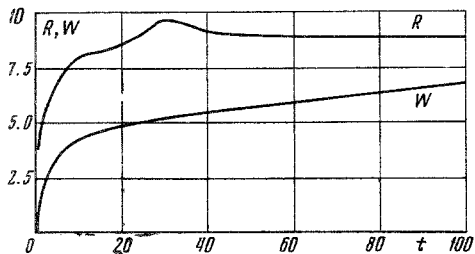


Fig. 9

aries were chosen by taking account of the nature of the change of the absorption coefficient with frequency. Comparisons with the calculations of the emission spectrum for various instants of time, carried out with subdivision into 655 spectral intervals (600 of these as in the tables [16] and 55 for quantum energies $\varepsilon > 19$ eV), showed that the error in the integrated characteristics, such as the radiation fluxes, did not exceed 10-20%.

The tables in [16] did not take account of the fine structure of the spectrum, as the absorption coefficient in them is averaged over intervals of $\Delta\nu = 250 \text{ cm}^{-1}$ (on the assumption of uniformity of the spectrum within the interval). We note that carrying through the calculations for the nonsteady-state gas-dynamic problem without averaging over spectral intervals is extremely difficult, in view of the limited capacity of the present-day computers.

Later, it is proposed to carry out the calculations using averaging over the true form of the spectrum, suggested in [12].

However, comparison of the results of the calculations for a different number of points and analysis of the form of the spectrum (at quite high pressures, the lines are strongly broadened, but the number which must be taken into account individually is small) give a basis for supposing that in the given case this does not lead to any noticeable difference in the results of the calculations.

Typical values of the parameters in these calculations are $T \approx 20,000^\circ\text{K}$ and $p \approx 3 \text{ atm}$, i.e., the electron density $N_e = 3 \cdot 10^{16} \text{ cm}^{-3}$ (degree of ionization $\alpha_e \approx 2$). Estimates of the times for the establishment of local thermodynamic equilibrium [2, 17] show that the use of the local thermodynamic equilibrium hypothesis as a first approximation in this range of thermodynamic parameters with characteristic times for the problem of order 10-100 μsec is quite reasonable.

In the tables used [3], no account was taken of the lowering of the ionization potential ΔI . Estimates [2, 17] show that in the given range of parameters $\Delta I \sim 2 \cdot 10^{-3} \text{ eV}$, i.e., negligibly small.

The quantities shown in the graphs are expressed in the following measurement units: pressure p in atm; radius r in mm; temperature T in 10^5°K ; density ρ in 10^{-5} g/cm^3 ; time t in μsec ; radiant flux density F in kW/cm^2 ; quantum energy $h\nu$ in eV; spectral flux density F_ν in $\text{kW/cm}^2 \cdot \text{eV}$; and emissivity W in %.

Figure 3 shows the temperature distribution with respect to radius for certain instants of time, and Fig. 4 demonstrates the behavior of temperature, pressure, and density with time at the center of the fireball. In the first instants, quite rapid cooling is observed of the hot core of the fireball down to a temperature of order $50,000^\circ\text{K}$. Then the cooling process becomes slower and the nature of the temperature profiles changes — temperature steps appear. The appearance of these steps, obviously, is related with the recombination process of nitrogen and oxygen ions, which is confirmed by the concentration of the primary components of air for characteristic values of $T = 45,000$ and $30,000^\circ\text{K}$ on the temperature profile at the instant $t = 5.17 \mu\text{sec}$.

$T \cdot 10^{-5}$	ρ	N^0	N^1	N^2	N^3
0.45	0.408	—	0.0368	4.3139	0.2116
0.30	0.725	0.0047	1.1026	0.4550	—
$T \cdot 10^{-5}$	O^0	O^1	O^2	O^3	
0.45	—	0.0200	0.3793	0.0197	
0.30	0.0020	0.3677	0.0493	—	

Here, N^i and O^i are the concentrations of the i -th multiply-ionized atoms of nitrogen and oxygen (concentrations are normalized to the undissociated state of air at $T = 293^\circ\text{K}$ and $\rho = \rho_0$).

When the temperature is increased from 30,000 to $50,000^\circ\text{K}$ and densities of order $\rho = 0.3 \cdot 10^{-5} \text{ g/cm}^3$, the process of second ionization of nitrogen and oxygen atoms takes place.

The steps at lower temperatures correspond to a sharp change of absorption coefficients of air at a temperature of order $20,000^\circ\text{K}$. We note that the scheme of calculation provides for the introduction of additional computational points along the radius, at locations of sharp change of temperature gradient.

The kinks in the temperature curves correspond to layers of gas spaced from the edge of the fireball at an optical distance

$$\tau = \int_r^R k(r) dr$$

of order unity. Here $k(r)$ is the average absorption coefficient (over the true spectrum at the given point r) for the "outgoing" radiation

$$k(r) = \int_0^\infty k_\nu(r) J_\nu d\nu / \int_0^\infty J_\nu d\nu \quad \left(J_\nu = \int_0^{\pi/2} I_\nu(\theta) \sin \theta d\theta \right)$$

where J_ν is the integrated intensity (with respect to angle) of the outgoing radiation.

It can be seen that even with considerable cooling of the fireball, it cannot be assumed to be completely transparent.

Figure 5 demonstrates the dependence of the optical thickness τ on the radius for the emergent radiation at certain selected instants of time. The optical thickness of the fireball initially increases somewhat with time, but then it falls again and becomes a value of order unity. During the whole time of cooling, the fireball from the explosion being considered is optically semitransparent. Comparison with Fig. 3 shows that the sharp increase of optical thickness along the radius corresponds to layers of air heated to a temperature of order 30,000°K, and the hot core of the fireball is almost transparent to radiation. For subsequent instants of time, the profile of the optical thickness becomes flatter, its value is reduced, and the spatial emission of the fireball begins.

Figure 6 shows the dependence on time of unidirectional flows of radiation at the center (upper curve) and at the edge of the fireball.

The presence of large fluxes at the center (especially in the first instants of time) possibly is one of the reasons for the absence of cooling waves. The theory of cooling waves [2] does not take account of these fluxes.

The spectral radiation fluxes F_ν , emerging from the surface of the fireball, are shown in Fig. 7 for certain selected instants of time and Fig. 8 shows the brightness temperatures T_e corresponding to them, found from the relation $F_\nu = \pi B(T_e)$. The relatively narrow frequency range of the emergent radiation should be noted. Quanta with $h\nu > 6.6$ eV are almost absent.

Figure 9 demonstrates the dependence of the radius of the fireball boundary R and the luminescence W on the time. For the radius of the fireball boundary R , we take the radius of the sphere bounding the region with $T > 2000^\circ\text{K}$. The nature of the relation for the boundary radius is associated with the change of pressure inside the fireball. At the instant $t \sim 30 \mu\text{sec}$, the pressure falls to a minimum value of order 0.8 atm, but the radius reaches its maximum value. Subsequently, atmospheric pressure is established and the radius of the boundary becomes almost constant. The fraction of the total explosion energy emitted from the surface of the fireball (luminescence) increases comparatively rapidly in the first instants until the temperature at the center of the region is quite high ($T_0 \sim 50,000^\circ\text{K}$). The growth of the luminescence then decreases markedly, which corresponds to the start of a slow process of spatial emission of the fireball. During cooling of the fireball for $t \sim 50 \mu\text{sec}$, the magnitude of the luminescence amounts in all to 6% of the total explosion energy.

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LITERATURE CITED

1. Yu. P. Raizer, "Breakdown and heating up of gases under the action of a laser beam," *Usp. Fiziol. Nauk*, **87**, No. 1, 29-63 (1965).
2. Ya. B. Zel'dovich and Yu. P. Raizer, *Physics of Shock Waves and High Temperature Hydrodynamic Phenomena*, 2nd Edition [in Russian], Nauka, Moscow (1966).
3. N. M. Kuznetsov, *Thermodynamic Functions and Shock Adiabats of Air at High Temperatures* [in Russian], Mashinostroenie, Moscow (1965).
4. *The Effect of Nuclear Weapons* [in Russian], Voenizdat, Moscow (1960).
5. *The Effect of a Nuclear Explosion. Collection of Translations*, Mir, Moscow (1971).
6. J. W. Daiber and H. M. Thomson, "Laser driven detonation waves in gases," *Phys. Fluids*, **10**, No. 6, 1162-1169 (1967).

7. S. H. Mead, "Plasma production with a multibeam laser system," *Phys. Fluids*, 13, No. 6, 1510-1518 (1970).
8. A. J. Alcock, E. Panarella, and S. A. Ramsden, Proceedings of the Seventh International Conference on Phenomena in Ionized Gases, Beograd, 224-227 (1966).
9. L. I. Sedov, *Methods of Similarity and Dimensionality in Mechanics* [in Russian], 5th Edition, Nauka, Moscow (1965).
10. H. L. Brode, "Numerical solutions of spherical blast waves," *J. Appl. Phys.*, 26, No. 6, 766-775 (1955).
11. D. E. Okhotsimskii, I. L. Kondrasheva, Z. P. Vlasova, and O. K. Kazakova, "Calculation of a point explosion taking account of counterpressure," *Trudy Matem. Inst. im. V.A. Steklova*, 50 (1957).
12. I. V. Nemchinov, "Averaged equations of radiation transport and their use for the solution of gas-dynamic problems," *Prikl. Matem. i Mekhan.*, 34, No. 4, 706-721 (1970).
13. H. Brode, "Review of nuclear weapon effects," *Ann. Rev. Nucl. Sci.*, 18, 153-202 (1968).
14. I. V. Nemchinov, "Some nonsteady-state problems of heat transfer by radiation" *Prikl. Matem. i Tekh. Fiz.*, 1, 36-57 (1960).
15. V. N. Kondrat'ev, I. V. Nemchinov, and V. M. Khazins, "Numerical calculation of the problem concerning the disintegration of a heated surface layer of a substance, taking account of stratification of its phase," *Prikl. Matem. i Tekh. Fiz.*, No. 4, 70-90 (1970).
16. I. V. Avilova, L. M. Biberman, V. S. Vorob'ev, V. M. Zamalin, G. A. Kobzev, A. N. Lagar'kov, A. Kh. Mnatsakanyan, and T. É. Norma, *Optical Properties of Hot Air* [in Russian], Nauka, Moscow (1970).
17. G. Grim, *Plasma Spectroscopy* [Russian translation], Atomizdat, Moscow (1969).